

Generalized dark gravity*

Tiberiu Harko[†]

*Department of Physics and Center for Theoretical and Computational Physics,
The University of Hong Kong, Pok Fu Lam Road, Hong Kong, P. R. China*

Francisco S. N. Lobo[‡]

*Centro de Astronomia e Astrofísica da Universidade de Lisboa,
Campo Grande, Ed. C8 1749-016 Lisboa, Portugal*

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Abstract

The late-time cosmic acceleration may be due to infra-red modifications of General Relativity. In particular, we consider a maximal extension of the Hilbert-Einstein action and analyze several interesting features of the theory. Generally, the motion is non-geodesic and takes place in the presence of an extra force, which is orthogonal to the four-velocity. These models could lead to some major differences, as compared to the predictions of General Relativity or other modified theories of gravity, in several problems of current interest, such as cosmology, gravitational collapse or the generation of gravitational waves. The study of these phenomena may also provide some specific signatures and effects, which could distinguish and discriminate between the various gravitational models.

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[†]Electronic address: harko@hkucc.hku.hk

[‡]Electronic address: flobo@cii.fc.ul.pt

Cosmology is said to be thriving in a golden age, where a central theme is the perplexing fact that the Universe is undergoing an accelerating expansion [1–3]. The latter, one of the most important and challenging current problems in cosmology, represents a new imbalance in the governing gravitational equations. Historically, physics has addressed such imbalances by either identifying sources that were previously unaccounted for, or by altering the governing equations. The cause of this acceleration still remains an open and tantalizing question. The standard model of cosmology has favored the first route to addressing the imbalance, namely, a missing energy-momentum component. In particular, the dark energy models are fundamental candidates responsible for the cosmic expansion (see [4] for a review).

One may also explore the alternative viewpoint, namely, through a modified gravity approach. It is widely believed that string theory is moving towards a viable quantum gravity theory. In this context, one of the key predictions of string theory is the existence of extra spatial dimensions. In the brane-world scenario, motivated by recent developments in string theory, the observed 3-dimensional universe is embedded in a higher-dimensional spacetime [5]. Most brane-world models, including those of the Randall-Sundrum type [6, 7], produce ultra-violet modifications to General Relativity, with extra-dimensional gravity dominating at high energies. However it is also possible for extra-dimensional gravity to dominate at low energies, leading to infra-red modifications of General Relativity. New features emerge in the brane scenario that may be more successful in providing a covariant infra-red modification of General Relativity, where it is possible for extra-dimensional gravity to dominate at low energies. The Dvali-Gabadadze-Porrati (DGP) models [8] achieve this via a brane induced gravity effect. The generalization of the DGP models to cosmology lead to late-accelerating cosmologies [9], even in the absence of a dark energy field [10]. This exciting feature of “self acceleration” may help towards a new resolution to the dark energy problem, although this model deserves further investigation as a viable cosmological model [11]. While the DGP braneworld offers an alternative explanation to the standard cosmological model, for the expansion history of the universe, it offers a paradigm for nature fundamentally distinct from dark energy models of cosmic acceleration, even those that perfectly mimic the same expansion history. It is also fundamental to understand how one may differentiate this modified theory of gravity from dark energy models. The DGP braneworld theory also alters the gravitational interaction itself, yielding unexpected phenomenological extensions beyond the expansion history. Tests from the solar system, large scale structure, lensing

all offer a window into understanding the perplexing nature of the cosmic acceleration and, perhaps, of gravity itself [11]. The structure formation [12] and the inclusion of inflation are also important requirements of DGP gravity, if it is to be a realistic alternative to the standard cosmological model.

In the context of modified theories of gravity, infra-red modifications to General Relativity have been extensively explored, where the consistency of various candidate models, have been analyzed (see [13] for a recent review). The Einstein field equation of General Relativity was first derived from an action principle by Hilbert, by adopting a linear function of the scalar curvature, R , in the gravitational Lagrangian density. However, there are no a priori reasons to restrict the gravitational Lagrangian to this form, and indeed several generalizations of the Einstein-Hilbert Lagrangian have been proposed, including “quadratic Lagrangians”, involving second order curvature invariants such as R^2 , $R_{\mu\nu}R^{\mu\nu}$, $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$, $\varepsilon^{\alpha\beta\mu\nu}R_{\alpha\beta\gamma\delta}R_{\mu\nu}^{\gamma\delta}$, $C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu}$, etc [14]. The physical motivations for these modifications of gravity were related to the possibility of a more realistic representation of the gravitational fields near curvature singularities and to create some first order approximation for the quantum theory of gravitational fields. In this context, a more general modification of the Einstein-Hilbert gravitational Lagrangian density involving an arbitrary function of the scalar invariant, $f(R)$, has been extensively explored in the literature. Recently, a renaissance of $f(R)$ modified theories of gravity has been verified in an attempt to explain the late-time accelerated expansion of the Universe [15].

In particular, a maximal extension of the Hilbert-Einstein action, has recently been explored [16], with the action given by

$$S = \int f(R, L_m) \sqrt{-g} d^4x, \quad (1)$$

where $f(R, L_m)$ is an arbitrary function of the Ricci scalar R , and of the Lagrangian density corresponding to matter, L_m . The energy-momentum tensor of matter is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \quad (2)$$

Thus, the gravitational field equation of $f(R, L_m)$ gravity model is given by

$$f_R(R, L_m) R_{\mu\nu} + (g_{\mu\nu} \nabla_\mu \nabla^\mu - \nabla_\mu \nabla_\nu) f_R(R, L_m) - \frac{1}{2} [f(R, L_m) - f_{L_m}(R, L_m) L_m] g_{\mu\nu} = \frac{1}{2} f_{L_m}(R, L_m) T_{\mu\nu}. \quad (3)$$

For the Hilbert-Einstein Lagrangian, $f(R, L_m) = R/2\kappa^2 + L_m$, we recover the standard Einstein field equations of General Relativity, i.e., $R_{\mu\nu} - (1/2)g_{\mu\nu}R = \kappa^2 T_{\mu\nu}$. For $f(R, L_m) = f_1(R) + f_2(R)G(L_m)$, where f_1 , f_2 and G are arbitrary functions of the Ricci scalar and of the matter Lagrangian density, respectively, we reobtain the field equations of modified gravity with an arbitrary curvature-matter coupling, considered in [17–19].

These models possess extremely interesting properties. First, the covariant divergence of the energy-momentum tensor is non-zero, and is given by

$$\nabla^\mu T_{\mu\nu} = 2\nabla^\mu \ln[f_{L_m}(R, L_m)] \frac{\partial L_m}{\partial g^{\mu\nu}}. \quad (4)$$

The requirement of the conservation of the energy-momentum tensor of matter, $\nabla^\mu T_{\mu\nu} = 0$, yields an effective functional relation between the matter Lagrangian density and the function $f_{L_m}(R, L_m)$, given by $\nabla^\mu \ln[f_{L_m}(R, L_m)] \partial L_m / \partial g^{\mu\nu} = 0$.

In second place, the motion of test particles is non-geodesic, and takes place in the presence of an extra force, orthogonal to the four-velocity. As a specific example, consider the case in which matter, assumed to be a perfect thermodynamic fluid, obeys a barotropic equation of state, with the thermodynamic pressure p being a function of the rest mass density of the matter ρ only, so that $p = p(\rho)$. In this case, the matter Lagrangian density, which in the general case could be a function of both density and pressure, $L_m = L_m(\rho, p)$, or of only one of the thermodynamic parameters, becomes an arbitrary function of the density of the matter ρ only, so that $L_m = L_m(\rho)$ (for more details, we refer the reader to [16, 20, 21]). Thus, the equation of motion of a test fluid in $f(R, L_m)$ gravity is

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda = f^\mu, \quad (5)$$

where

$$f^\mu = -\nabla_\nu \ln \left[f_{L_m}(R, L_m) \frac{dL_m(\rho)}{d\rho} \right] (u^\mu u^\nu - g^{\mu\nu}). \quad (6)$$

The extra-force f^μ is perpendicular to the four-velocity, u^μ , i.e., $f^\mu u_\mu = 0$.

The non-geodesic motion, due to the non-minimal couplings present in the model, implies the violation of the equivalence principle, which is highly constrained by solar system experimental tests [22, 23]. However, it has recently been argued, from data of the Abell Cluster A586, that the interaction between dark matter and dark energy implies the violation of the equivalence principle [24]. Thus, it is possible to test these models with non-minimal couplings in the context of the violation of the equivalence principle. It is also important

to emphasize that the violation of the equivalence principle is also found as a low-energy feature of some compactified versions of higher-dimensional theories.

An interesting application of the latter $f(R, L_m)$ gravity is the $f(R, T)$ model proposed in [25], where the action takes the following form

$$S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x. \quad (7)$$

$f(R, T)$ is an arbitrary function of the Ricci scalar, R , and of the trace T of the energy-momentum tensor of the matter, $T_{\mu\nu}$. L_m is the matter Lagrangian density. Note that the dependence from T may be induced by exotic imperfect fluids or quantum effects (conformal anomaly).

As a specific case of a $f(R, T)$ modified gravity model, consider $f(R, T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of T . In a cosmological setting, a simple model can be obtained by assuming a dust universe ($p = 0$, $T = \rho$), and by choosing the function $f(T)$ so that $f(T) = \lambda T$, where λ is a constant. Considering a flat Robertson-Walker metric, $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, the gravitational field equations are given by

$$3 \frac{\dot{a}^2}{a^2} = (8\pi + 3\lambda) \rho, \quad (8)$$

$$2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \lambda \rho, \quad (9)$$

respectively. Thus, this $f(R, T)$ gravity model is equivalent to a cosmological model with an effective cosmological constant $\Lambda_{\text{eff}} \propto H^2$, where $H = \dot{a}/a$ is the Hubble function. It is also interesting to note that generally for this choice of $f(R, T)$ the gravitational coupling becomes an effective and time dependent coupling of the form $G_{\text{eff}} = G \pm 2f'(T)$ (see [25] for more details). Thus the term $2f(T)$ in the gravitational action modifies the gravitational interaction between matter and curvature, replacing G by a running gravitational coupling parameter. The field equations reduce to a single equation for H ,

$$2\dot{H} + 3 \frac{8\pi + 2\lambda}{8\pi + 3\lambda} H^2 = 0, \quad (10)$$

with the general solution given by

$$H(t) = \frac{2(8\pi + 3\lambda)}{3(8\pi + 2\lambda)} \frac{1}{t}. \quad (11)$$

The scale factor evolves according to $a(t) = t^\alpha$, with $\alpha = 2(8\pi + 3\lambda)/3(8\pi + 2\lambda)$.

In conclusion, the predictions of the maximal extensions of General Relativity, namely the $f(R, L_m)$ gravity models could lead to some major differences, as compared to the predictions of standard General Relativity, or other generalized gravity models, in several problems of current interest, such as cosmology, gravitational collapse or the generation of gravitational waves. The study of these phenomena may also provide some specific signatures and effects, which could distinguish and discriminate between the various gravitational models. In order to explore in more detail the connections between the $f(R, L_m)$ gravity model and the cosmological evolution, some explicit physical models are necessary to be built.

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- [1] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], “Measurements of Omega and Lambda from 42 High-Redshift Supernovae,” *Astrophys. J.* **517**, 565 (1999).
 - [2] A. G. Riess *et al.* [Supernova Search Team Collaboration], “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant,” *Astron. J.* **116**, 1009 (1998).
 - [3] A. G. Riess *et al.* [Supernova Search Team Collaboration], “Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution,” *Astrophys. J.* **607**, 665 (2004).
 - [4] E. J. Copeland, M. Sami and S. Tsujikawa, “Dynamics of dark energy,” *Int. J. Mod. Phys. D* **15**, 1753 (2006) [hep-th/0603057].
 - [5] R. Maartens, “Brane-world gravity,” *Living Rev. Rel.* **7**, 7 (2004).
 - [6] L. Randall L and R. Sundrum, “A large mass hierarchy from a small extra dimension.” *Phys. Rev. Lett.* **83**, 3370 (1999) [arXiv:hep-ph/9905221].
 - [7] L. Randall and R. Sundrum, “An alternative to compactification,” *Phys. Rev. Lett.* **83**, 4690 (1999) [arXiv:hep-th/9906064].

- [8] G. Dvali, G. Gabadadze and M. Porrati, “4D gravity on a brane in 5D Minkowski space,” Phys. Lett. B **485**, 208 (2000).
- [9] C. Deffayet, “Cosmology on a brane in Minkowski bulk,” Phys. Lett. B **502**, 199 (2001).
- [10] R. A. Brown, R. Maartens, E. Papantonopoulos and V. Zamarias, “A late-accelerating universe with no dark energy - and a finite-temperature big bang,” JCAP **11** 008 (2005).
- [11] A. Lue, “The Phenomenology of Dvali-Gabadadze-Porrati Cosmologies,” astro-ph/0510068.
- [12] K. Koyama and R. Maartens, “Structure formation in the DGP cosmological model,” astro-ph/0511634.
- [13] S. 'i. Nojiri and S. D. Odintsov, “Unified cosmic history in modified gravity: from $F(R)$ theory to Lorentz non-invariant models,” Phys. Rept. **505**, 59 (2011) [arXiv:1011.0544 [gr-qc]].
- [14] F. S. N. Lobo, “The Dark side of gravity: Modified theories of gravity,” arXiv:0807.1640 [gr-qc].
- [15] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, “Is cosmic speed-up due to new gravitational physics?,” Phys. Rev. D **70**, 043528 (2004).
- [16] T. Harko and F. S. N. Lobo, “ $f(R, L_m)$ gravity,” Eur. Phys. J. C **70**, 373 (2010) [arXiv:1008.4193 [gr-qc]].
- [17] T. Harko, “Modified gravity with arbitrary coupling between matter and geometry,” Phys. Lett. B **669**, 376 (2008) [arXiv:0810.0742 [gr-qc]].
- [18] O. Bertolami, C. G. Boehmer, T. Harko and F. S. N. Lobo, “Extra force in $f(R)$ modified theories of gravity,” Phys. Rev. D **75**, 104016 (2007) [arXiv:0704.1733 [gr-qc]].
- [19] T. Harko, T. S. Koivisto and F. S. N. Lobo, “Palatini formulation of modified gravity with a nonminimal curvature-matter coupling,” Mod. Phys. Lett. A **26** (2011) 1467 [arXiv:1007.4415 [gr-qc]].
- [20] O. Bertolami, F. S. N. Lobo and J. Paramos, “Non-minimum coupling of perfect fluids to curvature,” Phys. Rev. D **78**, 064036 (2008) [arXiv:0806.4434 [gr-qc]].
- [21] O. Bertolami, J. Paramos, T. Harko and F. S. N. Lobo, “Non-minimal curvature-matter couplings in modified gravity,” arXiv:0811.2876 [gr-qc].
- [22] V. Faraoni, *Cosmology in Scalar-Tensor Gravity*, Kluwer Academic Publishers (2004).
- [23] O. Bertolami, J. Páramos and S. Turyshev, “General theory of relativity: Will it survive the next decade?,” gr-qc/0602016.
- [24] O. Bertolami, F. Gil Pedro and M. Le Delliou, “Dark Energy-Dark Matter Interaction and

- the Violation of the Equivalence Principle from the Abell Cluster A586,” Phys. Lett. B **654**, 165 (2007) [arXiv:astro-ph/0703462].
- [25] T. Harko, F. S. N. Lobo, S. 'i. Nojiri and S. D. Odintsov, “ $f(R, T)$ gravity,” Phys. Rev. D **84**, 024020 (2011) [arXiv:1104.2669 [gr-qc]].